In the Proceedings of the 1994 American Controls Conference in Baltimore, MD, June 1994.

Disk Drive Pivot Nonlinearity Modeling Part I: Frequency Domain

DANIEL ABRAMOVITCH
Hewlett-Packard Laboratories
1501 Page Mill Road, M/S 2U
Palo Alto, CA 94304
Phone: (415) 857-3806
Email: danny@hpldya.hpl.hp.com

FEEI WANG*
Hewlett-Packard Laboratories
1501 Page Mill Road, M/S 2U
Palo Alto, CA 94304
Phone: (415) 857-7807
Email: wangf@isl.stanford.edu

GENE FRANKLIN
Information Systems Laboratory
Stanford University
Stanford, CA 94305
Phone: (415) 723-4837
Email: franklin@isl.stanford.edu

Abstract– This paper describes studies done at HP Labs on the actuator pivot bearing nonlinearity of a small disk drive. The nonlinear frictional behavior of the pivot bearing varies with actuator position, from drive to drive, and with time and temperature. This nonlinear behavior has made the traditional linear disk drive models inadequate. Part I of the paper shows how the swept-sine/describing function method was used to develop a nonlinear model of the pivot. This model departs from the classical friction models, but does a good job of matching laboratory frequency domain measurements. Part II of this paper presents several additional models and describes time domain comparisons between the laboratory measurements and the model simulations.

1. Introduction

Friction in the actuator pivot of a small disk drive limits the low-frequency gain. This problem is much more pronounced in a small drive due to its small actuator inertia when compared with that of larger disk drives. Furthermore, such a small drive is designed for the mobile environment and thus will have to tolerate much more severe shock and vibration than typical in traditional disk drives. This translates to a requirement for additional gain at relatively low frequencies, where the shock and vibration play a more significant role.

The nonlinear frictional behavior of the pivot bearing varies with actuator position, from drive to drive, and with time and temperature. This nonlinear behavior has made the traditional linear disk drive models inadequate. However, it has been possible to make both time and frequency domain measurements both in the laboratory and in a nonlinear modeling package, namely SIMULINK[1]. In the frequency domain, this link was achieved by noting that a swept sine measurement – such as those taken with the HP 3562(3)A Control Systems Analyzer – measures the input/output describing function of the nonlinear system. If this measurement is exactly mimicked in the modeling package then a series of nonlinear models can be tested until the correct one is found. The search for the correct nonlinear model is continued until both time and frequency domains are matched between the model and the laboratory[2]. This nonlinear characterization should pave the way for a nonlinear adaptive controller that compensates for the effect of the nonlinearity. This paper will describe a small disk drive actuator pivot nonlinearity and how it is modeled using the swept-sine/describing function (SWS-DF) technique.

2. An Instrument Simulation

The target products for the high track density control algorithms are future disk drives. The best chance at understanding how these algorithms would work is by having a detailed model of the system. In order to verify that the models are based in reality, it is useful to verify them against existing versions of disk drives. Typically for a nonlinear system, measurements are done both in the time and the frequency domains. Part I of this paper deals with the frequency domain only. For reasons stated in [2], the swept-sine (also called sine-dwell) method[3, 4] was chosen over FFT based methods.

The plant frequency response was measured using the HP3562(3)A's swept-sine mode, the simulation implementation of which was developed by the authors in [2]. The swept-sine method essentially computes the describing function of a nonlinear system. This is important for several reasons.

- It allows the user to characterize system nonlinearities.
- It allows the user to verify a nonlinear model against measurements of the laboratory system.
- It allows the user to predict effects of a nonlinearity on the overall control loop using a verified model.

*Feei Wang is a Ph.D. candidate at the Information Systems Lab and is doing joint research at Hewlett-Packard Labs under HPL's Intern program.

3. The Disk Drive Actuator Pivot Nonlinearity

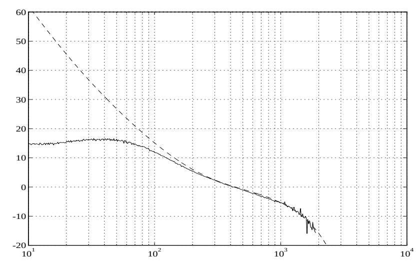

Figure 1: The Actuator Pivot Nonlinearity

The effects of the disk drive actuator pivot nonlinearity are shown in Figure 1. The dashed line shows an idealized open loop frequency response. The solid line shows a measurement made on a small drive. As there is nothing in the arm structure that should give this type of a resonant behavior at low frequency, one might surmise that this "low frequency pole" is actually due to some nonlinear behavior in the pivot bearing *i.e.,* friction. (Further measurements have verified this.)

One of the advantages of a block diagram based model is that it allows us to rapidly alter the model to match a new type of measurement. The laboratory setup was altered in order to focus on the pivot itself. Rather than using the drive power amplifier, voltage was applied directly across the actuator coils. Rather than reading a relative position from servo marks on the disk, the read/write head position was measured with a Laser Doppler Vibrometer (LDV). The simplified model that corresponds to this laboratory setup is shown in Figure 4. The objective was to find both the form of the feedback and the parameters of the nonlinear model such that the model's frequency response would match the measured system's frequency response. One of the key features of an acceptable model is that the model "measurements" must change with the input amplitude just as the laboratory measurements do.

3.1 Preload Velocity Feedback

A natural starting point for a nonlinear model is the so called preload nonlinearity feeding back from the velocity. The preload nonlinearity is comprised of Coulomb friction plus viscous damping[5]. Figure 2 shows a comparison between the laboratory measurement of the mechanical transfer function and the model simulation result[1]. While the match appears

[1]The simulated frequency response curve shown is not smooth at low frequencies because noise was injected into the model. This noise was removed in later models so averaging need not be done. This greatly reduces simulation time.

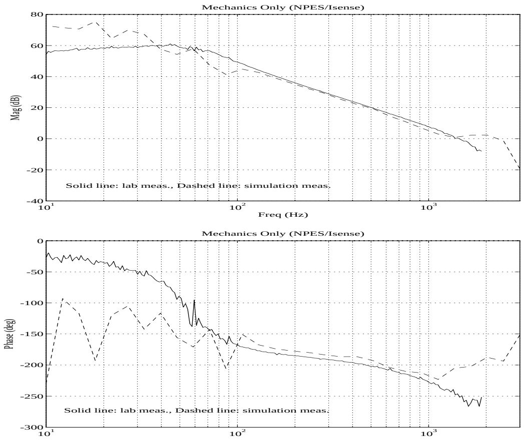

Figure 2: Matching the model: The arm mechanics model includes a single preload as its nonlinear component. Solid line: lab measurement, Dashed line: simulation measurement. The mismatch points to the need for a position feedback term.

good at frequencies above 100 Hz, the match deteriorates below this and the slope of the model frequency response (on average) appears closer to a 20 dB/decade slope than the nearly flat line of the laboratory measurement. The reason for this is clear if we replace the nonlinear element with simply viscous (linear) damping. In that case, the transfer function is given by

$$\frac{X(s)}{R(s)} = \frac{1}{J}\frac{1}{s(s+a)}, \qquad \text{where } a \text{ is the viscosity} \tag{1}$$

which has a 20 dB/decade slope at low frequency. In order to get the 0 dB/decade behavior seen in the lab measurement, it is necessary to add a position feedback term.

3.2 The Two Preload Model

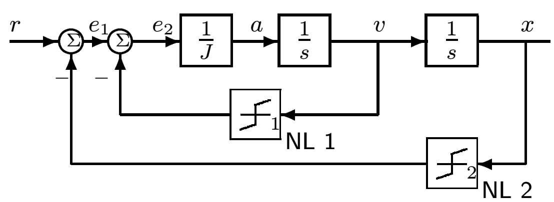

Figure 3: Two "preload" model of pivot nonlinearity. Although this allows for some interesting qualitative predictions, model measurements based upon this do not match laboratory measurements.

A good starting point for a model that contains both position and velocity feedback nonlinearities is shown in Figure 3. A linear position dependence is not sufficient because there will be no variation in low-frequency gain as input amplitude varies. The two preload model is simple enough that some describing function analysis can still be done qualitatively.

An input signal is injected at r and measurements of a, v, and/or x are possible. Describing function analysis assumes that the system can be approximated as operating around the single sinusoid injected into the reference. In this case, the reference sinusoid is $r(t)$ and the signal that the other responses will be defined in reference to will be that of the velocity, $v(t)$.

$$r(t) = R\sin(\omega t), \tag{2}$$

$$v(t) = A_v\cos(\omega t), \tag{3}$$

$$x(t) = A_x\sin(\omega t), \tag{4}$$

$$a(t) = -A_a\sin(\omega t). \tag{5}$$

where the amplitudes can be given as

$$A_x = \frac{A_v}{\omega}, \tag{6}$$

$$A_v = A_v, \tag{7}$$

$$A_a = \omega A_v. \tag{8}$$

There are two nonlinearities in the system. The nonlinear velocity feedback is defined as

$$NL_1 = K_c\mathrm{sgn}(v) + K_v v, \tag{9}$$

where K_c is the Coulomb level and K_v is the viscosity. Its describing function is given by

$$DF_1 = \frac{4K_c}{\pi A_v} + K_v v. \tag{10}$$

Likewise the nonlinear position feedback is defined as

$$NL_2 = D\mathrm{sgn}(x) + mx, \tag{11}$$

where m is the linear spring term and D is the signum spring level. Its describing function is similarly

$$DF_2 = \frac{4D}{\pi A_x} + mx \tag{12}$$

Under the assumption that the signals are mostly sinusoidal, it is possible to combine these describing functions with the linear portion of the system to obtain:

$$\frac{X(s)}{R(s)} = \frac{1}{J}\cdot\frac{1}{s^2 + \left(\frac{4K_c}{\pi A_v J} + \frac{K_v}{J}\right)s + \left(\frac{4D}{\pi A_x J} + \frac{m}{J}\right)} \tag{13}$$

This then is a "describing function" model of the above system with the two preload nonlinearities. This model can be used to qualitatively predict some behaviors of the system.

One of the features of swept-sine measurements on the HP 3562(3)A is the ability to have the input signal amplitude adjusted during the sweep so that the amplitude of one of the measured output signals is kept constant. This is known as autogaining. In the laboratory instrument, it is sometimes difficult to obtain data in this fashion due to the effects of the nonlinear behavior on the autogaining algorithm. However, the same concept is fairly useful in analyzing the behavior of this model. For example if the amplitude A_v is controlled as the frequency is swept, then the amplitude of A_x can also be determined from the relationship $A_x = \frac{A_v}{\omega} = \frac{jA_v}{s}$. In this case, the transfer function from $R(s)$ to $X(s)$ is given by:

$$\frac{X(s)}{R(s)} = \frac{1}{J}\cdot\frac{1}{s^2 + (a_1+\tilde{a}_1)s + (a_2+\tilde{a}_2)} \tag{14}$$

where

$$\begin{aligned} a_1 &= \tfrac{K_v}{J} & \tilde{a}_1 &= \tfrac{4(K_c - jD)}{\pi A_v J} \\ a_2 &= \tfrac{m}{J} & \tilde{a}_2 &= 0 \end{aligned} \tag{15}$$

Likewise, if the amplitude A_x is controlled, then the amplitude A_v can also be determined from $A_v = \omega A_x = -jsA_x$. In this case the transfer function from $R(s)$ to $X(s)$ is given by:

$$\frac{X(s)}{R(s)} = \frac{1}{J}\cdot\frac{1}{s^2 + \left(b_1+\tilde{b}_1\right)s + \left(b_2+\tilde{b}_2\right)} \tag{16}$$

where

$$\begin{aligned} b_1 &= \tfrac{K_v}{J} & \tilde{b}_1 &= 0 \\ b_2 &= \tfrac{m}{J} & \tilde{b}_2 &= \tfrac{4(jK_c + D)}{\pi A_x J} \end{aligned} \tag{17}$$

These models are useful since they can be directly compared with the classic spring-mass-damper model:

$$\frac{X(s)}{R(s)} = \frac{1}{J}\cdot\frac{1}{s^2 + 2\zeta\omega_n s + \omega_n^2}. \tag{18}$$

Several things are worth noting here:

1) The nonlinear effect is inversely proportional to A_v, A_x, and J.

2) If the amplitude A_v (velocity) is held constant throughout a single measurement then

   - $a_2 = \omega_n^2$ (the undamped natural frequency) is constant,
   - $a_1 + \tilde{a}_1 = 2\zeta\omega_n$ changes, and
   - as A_v goes up $\tilde{a}_1$ drops so ζ (the damping ratio) drops.

3) If the amplitude A_x (position) is held constant throughout a single measurement then

   - $a_1 = 2\zeta\omega_n$ is constant,
   - $a_2 + \tilde{a}_2 = \omega_n^2$ changes, and
   - as A_x goes up $\tilde{a}_2$ drops so ω_n drops and ζ goes up.

4) It is difficult to analyze what happens when the amplitude R (reference) is held constant through a single measurement, but one could speculate that some combination of the above two behaviors would be observed.

This model is extremely useful to qualitatively understand what should be observed from laboratory measurements of the system. It is also analytically tractable. There are problems however. The first is that preload model of nonlinear spring is not physical. Secondly, simulations of this model do not quantitatively match our lab measurements, so this model was not pursued further.

3.3 Preload Velocity Feedback Plus Two Slope Position Feedback

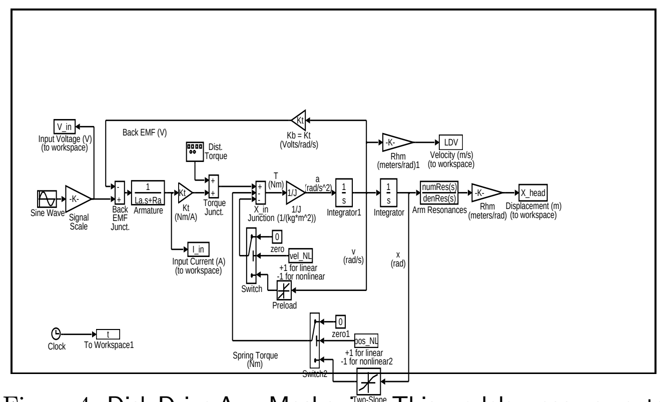

Figure 4: Disk Drive Arm Mechanics. This models measurements made by putting voltage directly into the coils. Sensing done with a Laser Doppler Vibrometer (LDV). The system is open-loop *i.e.,* there is no track following.

In an attempt to improve the correspondence with laboratory measurements several position feedback (nonlinear spring) models were tried. Among them was a spring constant that saturated. However, further laboratory measurements pointed the way towards a different multiple slope spring. The effective spring constant, calculated from the spring lines of frequency response plots taken at different input amplitudes, showed a two-slope behavior.

The nonlinear mechanical model is shown in Figure 4, where the feedback from velocity is a standard Coulomb friction and viscous damping term and the feedback from position is a two-slope spring. The results in Figure 5 show that an excellent match has been achieved between laboratory measurements and model measurements. Work has also been done to achieve matching in the time domain. Results on this work are presented in Part II of this paper[6].

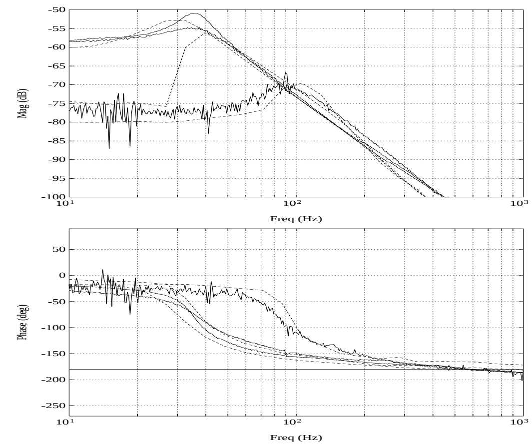

Figure 5: Matching the Model: Arm Mechanics Only. Solid line: lab measurement, Dashed line: simulation measurement. 2 mV, 5 mV, & 10 mV input to coils in both lab and simulation.

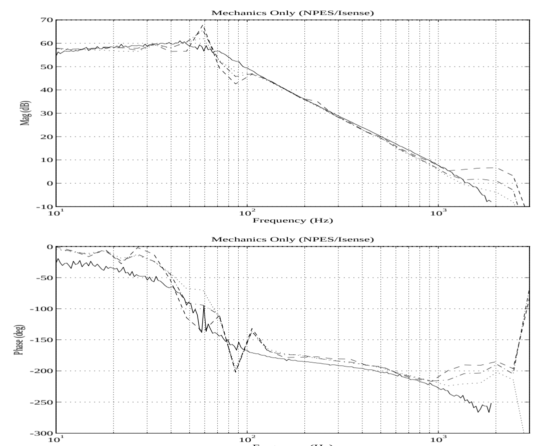

Figure 6: Matching the Model: Mechanics only, with once around operation. Solid line: lab measurement, Dashed line: simulation measurement. 5 mV, 10 mV, & 20 mV input to TF_{in} in simulation. 10 mV input to TF_{in} in lab.

4. Using the Nonlinear Model

Given that a reasonable match has been achieved in the frequency domain, the question arises in how to use this mechanics model. One idea is to use the improved mechanics model in an overall model of the system that includes the periodic motion caused by the actuator following the spindle eccentricities – commonly called *once around*. The mechanics model must be adjusted to account for the effect of the power amplifier. The model, complete with elements such as the power amplifier and the digital compensator, allows us to make some predictions about how the closed-loop system will behave. These can later be verified by direct measurement.

Earlier measurements of the system running in closed-loop revealed that the pivot nonlinearity showed up in the mechanics only, closed-loop, and open-loop responses. Furthermore, this nonlinear behavior seemed to be independent of the swept-sine input amplitude. This was particularly confusing since one expects that amplitude dependent nonlinearities will show some *amplitude dependence*. This apparent paradox was resolved using the model in closed-loop simulations. It was observed that not only did the nonlinearity show up in the mechanics only, closed-loop, and open-loop responses, but the once around (simulating the motion of following spindle eccentricities) acts as a 90 Hz "dither" to alter the nonlinear system's operating point. Effectively this keeps the system in a high input amplitude region of nonlinearity. Thus, the closed-loop system becomes relatively less sensitive to variations of the swept-sine input amplitude. This is shown in Figures 6

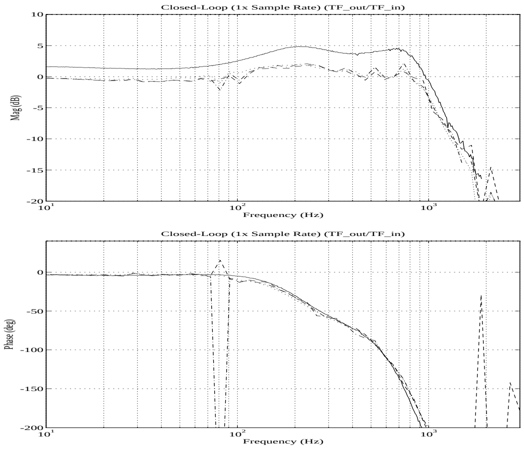

Figure 7: Matching the Model: Closed-loop, with once around operation. Solid line: lab measurement, Dashed line: simulation measurement. 5 mV, 10 mV, & 20 mV input to TF_{in} in simulation. 10 mV input to TF_{in} in lab.

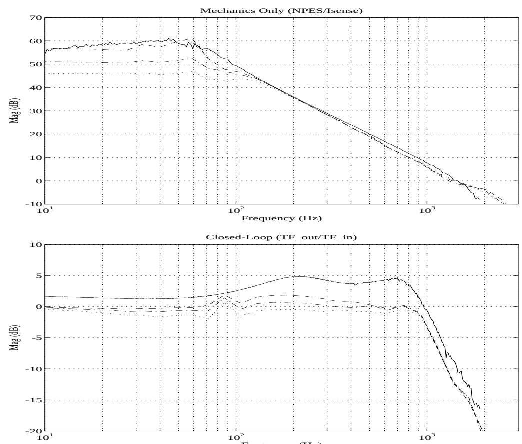

Figure 8: Using the model: changing nonlinear parameters affects closed-loop response. In this case the upper spring slope of the position dependent nonlinearity is altered. Once around is being followed. The lines correspond to 30 Hz, 60 Hz, & 90 Hz "high amplitude" resonance.

– 7. Figure 6 shows a measurement of the system mechanics while the once-around is being followed. Figure 7 shows laboratory measurements and model simulations of the closed-loop system dynamics. Note that it appears that there is a gain difference of approximately 2 dB at low frequency between the measurement and simulation results. This is likely due to some amplifiers in the laboratory testbed that were not properly accounted for in the system model. Note that although the once-around desensitizes the measurements to input amplitude variations, variations in the nonlinear parameters should show up in the responses. This is verified in Figure 8.

Finally, one could predict that if the open-loop measurements of Figure 5 were repeated with a 90 Hz signal summed into the coils of the arm only testbed, then these measurements would be desensitized to swept-sine amplitude variations. An experiment was done that verified this prediction, however the data is not presented here.

5. Summary

The swept-sine describing function method described previously [2] has been used to characterize the nonlinear dynamics of a small disk drive. A key discovery is that the frictional behavior is not determined soley by velocity feedback or position feedback, but by a combination of the two acting at all times. This is in contrast to the classical preload (Coulomb plus viscous) model of friction[5] which assumes only velocity dependence. Conversely, the work done by Dahl and others found a friction model for ball bearings that depends only on position[7, 8, 9]. However, many of these measurements were made with the velocity held constant. In effect by holding velocity constant, it is impossible to study velocity dependence. By allowing both position and velocity dependence, this model has produced a closer match to laboratory measurements than either of these constructs on their own.

As it is, this model does allow quite a few predictions to be made about the actual and future systems. These predictions can be tested in the frequency domain on the model by using the swept-sine/describing function method. Of course, the describing function generated by the swept-sine measurement does throw away data about the higher harmonics of the response. Thus, while it is extremely useful for characterizing the actuator pivot bearing friction, it is incomplete. Part II of this paper discusses comparisons done with hysteresis loops in the time domain[6] and reconciling these with the results here.

6. Acknowledgements

The results discussed in this paper are the result of a team effort at HP Labs. Frequency and time domain laboratory measurements were made by Dick Henze and Terril Hurst.

References

[1] The MathWorks, Inc., Sherborn, MA, *SIMULINK: A Program for Simulating Dynamic Systems*, 1992.

[2] F. Wang, D. Abramovitch, and G. Franklin, "A method for verifying measurements and models of linear and nonlinear systems," in *Proceedings of the 1993 American Control Conference*, (San Francisco, CA), pp. 93–97, AACC, IEEE, June 1993.

[3] Hewlett-Packard, *Control System Development Using Dynamic Signal Analyzers: Application Note 243-2*, 1984.

[4] Hewlett-Packard, *HP 3563A Control Systems Analyzer*, 1990.

[5] A. Gelb and W. E. Vander Velde, *Multiple-Input Describing Functions and Nonlinear System Design*. New York: McGraw-Hill Book Company, 1979.

[6] F. Wang, T. Hurst, D. Abramovitch, and G. Franklin, "Disk drive pivot nonlinearity modeling Part II: Time Domain," in *Proceedings of the 1994 American Control Conference*, (Baltimore, MD), AACC, IEEE, June 1994.

[7] P. R. Dahl, "A solid friction model," Technical Report TOR-158(3107-18), The Aerospace Corporation, El Segundo, CA, May 1968.

[8] P. R. Dahl, "Measurement of solid friction parameters of ball bearings," in *Proceedings of the 6th Annual Symposium on Incremental Motion, Control Systems and Devices*, pp. 49–60, University of Illinois, AIAA, 1977.

[9] P. R. Dahl, "Solid friction damping of mechanical vibrations," *AIAA Journal*, vol. 14, pp. 1675–1682, December 1976.